Closing Wed: HW_2A,2B (5.3,5.4)
Closing Thurs: HW_2C (5.5)
Monday is a holiday! (no office hours)

Quick review:

Def'n: The "signed" area between $f(x)$ and the x-axis from $x = a$ to $x = b$ is the *definite integral*:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

FTOC(2): If F(x) is <u>any</u> antideriv. of $f(x)$,

$$\int_a^b f(x)dx = F(b) - F(a)$$

Entry Task: Evaluate

$$\int_0^4 e^x + \sqrt{x^3}\,dx$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: The **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$. We write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is any antideriv. of $f(x)$.

Example:

$$\int \frac{4}{x^2} + \sec^2(x) + \frac{5}{x^2+1}\ dx$$

Net Change and Total Change

The FTOC(2) says the **net change** in $f(x)$ from $x = a$ to $x = b$ is the integral of its **rate**. That is:

$$\int_a^b f'(t)dt = f(b) - f(a)$$

For example: Assume an object is moving along a straight line (up/down or left/right).

$s(t)$ = 'location at time *t*'

$v(t)$ = 'velocity at time t'

pos. $v(t)$ means moving up/right

neg. $v(t)$ means moving down/left

The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

'integral of velocity'= '**net change** in dist'

We also call this the *displacement*.

We define **total change** in dist. by

$$\int_a^b |v(t)| dt$$

which we compute by

1. Solving $v(t) = 0$ for t.

2. Splitting up integral.
   Compute positive and negative areas separately.

3. Adding together as positive numbers.

Example: $v(t) = t^2 - 2t - 8$ ft/sec
Compute the total distance traveled from t = 1 to t = 6.

5.5 The Substitution Rule

Motivation:

1. Find the following derivatives

Function	Derivative?
$\sin(x^4)$	
$\mathrm{e}^{\tan(x)}$	
$\ln(\mathrm{x}^4+1)$	

2. Rewrite as integrals:

$$\int \qquad dx = \sin(x^4) + C$$

$$\int \qquad dx = \mathrm{e}^{\tan(x)} + C$$

$$\int \qquad dx = \ln(\mathrm{x}^4+1) + C$$

3. Guess and check the answer to:

$$\int 7x^6 \sin(x^7)\, dx =$$

Observations:

1. We are reversing the “chain rule”.
2. In each case, we see
   “inside” = a function inside another
   “outside” = derivative of inside

To help us mechanically see these connections, we use what we call:

The Substitution Rule:
If we write u = g(x) and du = g’(x) dx, then

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)du$$

Aside (you do not need to write this)

Some theory

Recall:

$$\int_a^b f(g(x))g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(g(x_i))g'(x_i)\Delta x$$

If we replace u = g(x), then we are "transforming" the problem from one involving *x* and *y* to one with *u* and *y*.

This changes ***everything*** in the set up. The lower bound, upper bound, widths, and integrand all change!

Recall from Math 124 that

$$g'(x) = \frac{du}{dx} \approx \frac{\Delta u}{\Delta x}$$

(with more accuracy when Δx is small)

Thus, we can say that

$$g'(x)\Delta x \approx \Delta u$$

In other words, if the width of the rectangles using *x* and *y* is Δx, then the width of the rectangles using *u* and *y* is $g'(x)\Delta x$.

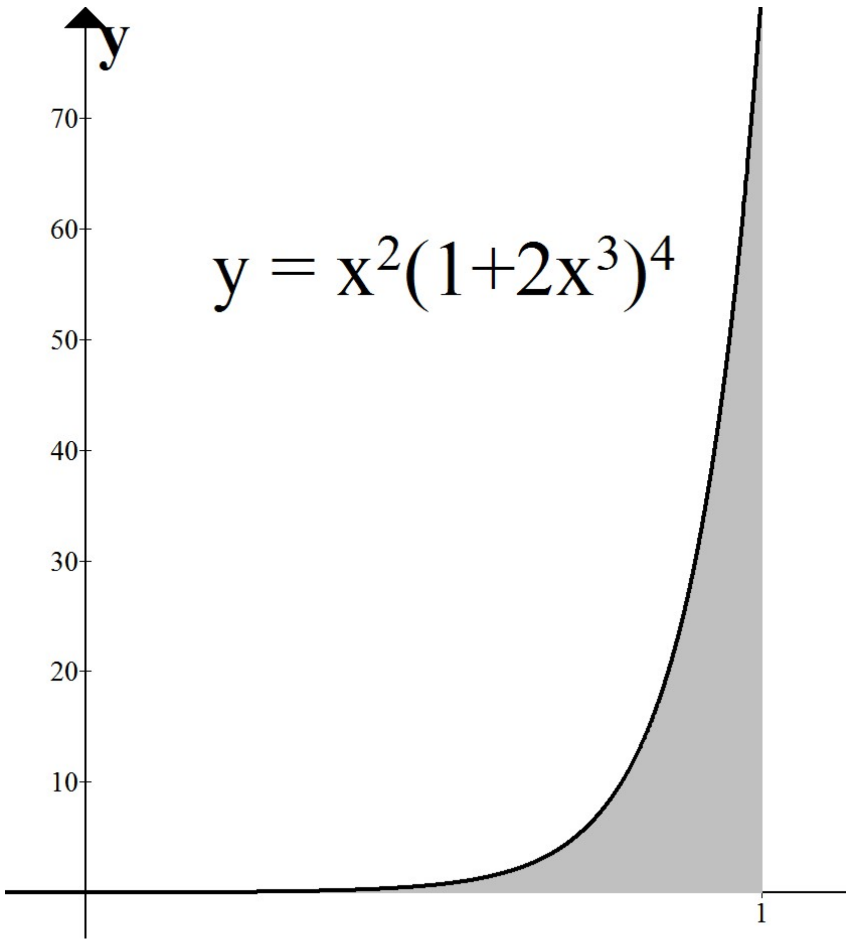
And if we write $u_i = g(x_i)$, then

$$\int_a^b f(g(x))g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(g(x_i))g'(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f(u_i)\Delta u$$

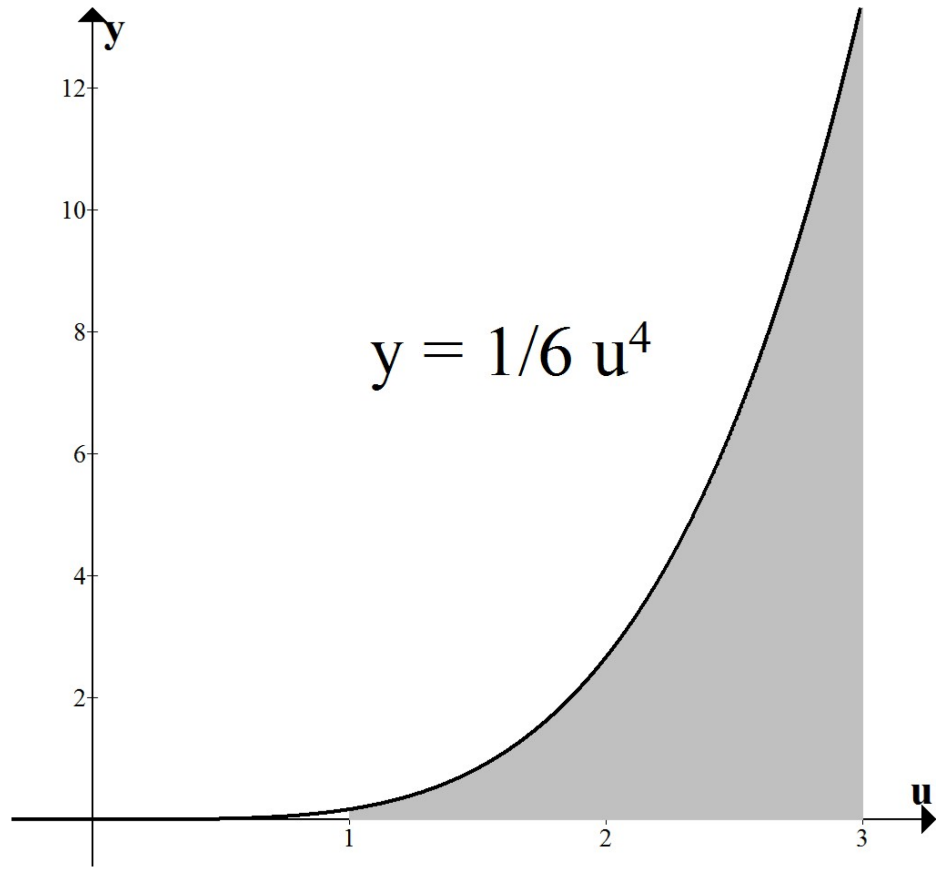
$$= \int_{g(a)}^{g(b)} f(u)du$$

Here is a visual example of this transformation



$$\int_0^1 x^2(1+2x^3)^4 dx$$

Using u = 1 + 2x^3 and du = 6x^2 dx, we get



$$\int_1^3 \frac{1}{6} u^4 du$$

Examples:

First, try u = “inside function”

1. $\int x^4(1+x^5)^{31}\,dx$

2. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

3. $\displaystyle\int_2^3 x^2 e^{x^3}\,dx$

4. $\displaystyle\int \frac{x\sin(x^2)}{\cos^2(\cos(x^2))}\,dx$

Examples:

Then, try u = “denominator”

1. $$\int_0^1 \frac{x}{x^2+3}\, dx$$

2. $$\int \tan(x)\, dx$$

What to do when the “old” variable remains:

Examples:

1. $\int x^3\sqrt{2+x^2}\,dx$

2. $\int \frac{\mathrm{x}^7}{x^4+1}\,dx$